Knowledge Space Theory, Formal Concept Analysis and Computerized Psychological Assessment.

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Abstract

In the present study the use of Knowledge Space Theory (KST), jointly with Formal Concept Analysis (FCA), is proposed to develop a formal representation of the relations between the items of a questionnaire and a set of psychodiagnostic criteria. This formal representation can be used to develop an efficient adaptive tool for psychological assessment. Rusch and Wille (1996) show some interesting connections between KST and FCA; these connections are applied to construct knowledge structures starting from a formal context representing the relations between items and criteria. The proposed general methodology has been applied, as an example, to the Maudsley Obsessive-Compulsive Questionnaire. We used a data-set provided by a sample of patients with a diagnosis of Obsessive-Compulsive Disorder to validate the obtained structures. The parameters of the Basic Local Independence Model (BLIM) have been estimated for the obtained knowledge structures. The fit of each model has been tested by parametric bootstrap because of the sparseness of the derived data matrix. Results are discussed in light of both psychological and methodological relapses. In particular we propose a re-interpretation of the BLIM parameters that seems suitable for testing reliability and construct validity; furthermore it is pointed out how the obtained structures could represent the starting point for the development of a computerized assessment tool.

Keywords: Knowledge Space Theory, Formal Concept Analysis, Psychological Assessment, Reliability, Construct Validity.

Introduction

In the present study Knowledge Space Theory has been applied, jointly with Formal Concept Analysis, in the definition of a methodology for constructing a formal representation of the relation between the items of a given questionnaire. The obtained formal representation can be used to develop an adaptive and efficient tool for psychological assessment. An application of this methodology is presented regarding the items of the Maudsley Obsessive-Compulsive Questionnaire.

Knowledge Space Theory (KST; Doignon & Falmagne, 1985, 1999) is traditionally applied for the efficient computerized assessment of knowledge and training (Doignon & Falmagne, 1999; Dowling, Hockemeyer, & Ludwig, 1996; Hockemeyer, Held, & Albert, 1998), therefore the main field of application of this theory is education. The computerized procedures built using this theoretical background provide evaluations, in the form of attitudes outlines, showing what the subject is able to do and what he is ready to learn. One aim of this paper is to identify how these useful aspects of KST can be applied to psychological assessment (e.g. clinical assessment, personality assessment, etc.). The first aim of this paper is to give an answer to this question. At least two main aspects have to be considered:

A) the main concepts of the theory have to be reinterpreted in light of this field of application and it has to be verified whether this "translation" is allowed or not;

B) the theoretical hypothesized models have to be tested to determine whether they provide a good representation of the empirical data.

The main concepts of KST are described below. A *knowledge domain* can be defined as the set Q of all the items that it is possible to investigate about a specific topic. The main authors of KST have focused their attention on topics included in the areas of Mathematics and Statistics, even if they did not exclude the possibility to apply the logical and formal structure of KST to different topics (Doignon & Falmagne, 1999). Some further explanations will clarify both the reason why KST has been mostly applied in those specific contexts, and why it has been used in this study.

Given the knowledge domain Q, a *knowledge state* $K \subseteq Q$ represents the subset of Q that a specific subject is able to solve. A *knowledge structure* $\mathcal{K}$ is then defined as a collection of knowledge states including at least the empty set ($\emptyset$) and the total set (Q). In the traditional formal notation a knowledge structure is denoted as $(Q, \mathcal{K})$ where Q represents the knowledge domain and $\mathcal{K}$ represents the collection of subsets included in the structure. The knowledge structure is a representation of the implications among the items belonging to Q. Using this notation it is possible to identify (i) the field of knowledge that is under consideration, (ii) the specific knowledge presented by a single subject and (iii) the relations that link together the different items of Q.

An example may be useful to better understand this last sentence: consider the following knowledge structure defined on a set Q of three items a, b and c:

$$\mathcal{K} := \{\emptyset, \{a\}, \{b\}, \{a, c\}, \{a, b, c\}\} \tag{1}$$

In the knowledge structure $\mathcal{K}$ we can observe that the knowledge domain Q is composed by the three items a, b and c; the relations among the items in Q determine the admissible knowledge states. In the given example the mastery of item a is a prerequisite for the mastery of item c (i.e. a is a prerequisite for c) in fact there is no state in $\mathcal{K}$ containing c and not containing a. In other words any subject failing item a would necessarily fail item c (excluding the possibility of careless errors and lucky guesses).

When the collection $\mathcal{K}$ of knowledge states of a knowledge structure $(Q, \mathcal{K})$ is closed under union (i.e. every union of knowledge states is again a knowledge state included in the structure; formally: $\bigcup \mathcal{F} \in \mathcal{K}$ for all $\mathcal{F} \subseteq \mathcal{K}$), the knowledge structure is then called a *knowledge space*.

An interesting property of a knowledge space is that more than a single set of prerequisites are allowed for an item. This means that the same item can be solved using different solution strategies (Doignon & Falmagne, 1985).

Another fundamental concept of KST is that of a *skill-map* (Albert, Schrepp, & Held, 1992; Doignon & Falmagne, 1999; Falmagne, Koppen, Villano, & Doignon, 1990; Hockemeyer, Conlan, Wade, & Albert, 2003; Lukas & Albert, 1993). In our view this concept can be easily adapted and reinterpreted in a clinical context. A skill map is a triple (Q, S, f) where Q is a non-empty set of items, S is a non-empty set of skills and f is a mapping from Q to $2^S \setminus \{\emptyset\}$ (i.e. the powerset of S excluding the empty-set; Doignon & Falmagne, 1999). For any item q in Q the subset $f(q)$ of S represents the set of skills assigned to q. In other words it means that if a subject solves item q he has all the skills included in $f(q)$ (*conjunctive model*). In this article we use the concept of a skill-map and adapt it to the psychological assessment context where the elements of S are interpreted as clinical symptoms rather than abilities needed to solve a specific item.

In the section "Methods" a procedure for deriving a knowledge structure from a skill-map is discussed. This procedure is essentially based on some interesting connections between KST and another theory, called Formal Concept Analysis (FCA; Ganter & Wille, 1999; Wille, 1982), pointed out by Rusch and Wille (1996).

Before starting the presentation of the specific aspects of the present study, it seems necessary to present some basic notions about Formal Concept Analysis in order to allow a better comprehension of its joint application with KST.

The first basic notion of FCA is the *formal context* defined as a triple (G, M, I) where G is a set of *objects*, M is a set of *attributes* and I is a binary relation between the set of objects and the set of attributes. A formal context is usually represented by a Boolean matrix where each row is an object while each column is an attribute. Whenever a value 1 is present in the entry (g, m) it means that the relation gIm holds, in other words it means that the object g has the attribute m. Between the objects and the attributes of a formal context a *Galois connection* is defined. For all the sets $A \subseteq G$ and $B \subseteq M$, the following two transformations define the Galois connection:

$$A' := \{m \in M \mid gIm, \forall g \in A\} \tag{2}$$

$$B' := \{g \in G \mid gIm, \forall m \in B\} \tag{3}$$

In words, A' is the collection of all the attributes that all the objects in A have in common. Dually B' is the collection of all the objects that possess all the attributes in B. It is now possible to introduce a fundamental notion of FCA. The pair (A, B) is called a *formal concept* if it satisfies the following two conditions: $A = B'$ and $B = A'$. The so called *extent* A of the formal concept contains exactly those objects of G that have all the attributes in B; the so called *intent* B of the formal concept includes exactly those attributes satisfied by all the objects in A.

A sub-concept super-concept relation is then defined in the following way:

$$(A_1, B_1) \leq (A_2, B_2) \Leftrightarrow A_1 \subseteq A_2 \tag{4}$$

or equivalently

$$(A_1, B_1) \leq (A_2, B_2) \Leftrightarrow B_1 \supseteq B_2 \tag{5}$$

In words, a concept is of a lower level when it has a larger extent (or equivalently a smaller intent).

The concepts of a context form a *complete lattice* (Birkhoff, 1937, 1967) that is called the *concept lattice* of (G, M, I). The intents of a concept lattice are closed under intersection i.e. each intersection of sets of attributes is included in the lattice. Rusch and Wille (1996) show that the collection of the complements of the intents of a formal context is closed under set-union and so it is a knowledge space (Rusch & Wille, 1996). In their article, the authors start from a formal context defined by the set G of subjects (that in this case are treated as formal objects), the set M of items, and the binary relation gIm meaning that the subject g has correctly answered item m. In these terms a response pattern can be seen as a set of formal attributes, an intent. By the relation between the intents of a formal context and their complements, the authors derive a so called *knowledge context* having the domain defined by the set of items and the states by the complements of the observed response patterns. Using this methodology it is then possible to construct a knowledge space starting from a formal context.

Concerning point B, earlier introduced in this section, an example of the joint application of KST and FCA in a clinical context, similar to the one introduced in Spoto et al. (2008), is presented. In this application three knowledge structures representing the starting point for the construction of an adaptive computerized assessment tool are built and empirically tested. Such structures represent the relations between the items of the Maudsley Obsessive-Compulsive Questionnaire (MOCQ; Hodgson & Rachman, 1977) for Obsessive-Compulsive Disorder (OCD) included in the Diagnostic and Statistical Manual of Mental Disorders IV-TR (DSM-IV-TR; American Psychiatric Association [APA], 1995).

The results of this study have to be considered as a mere example of the potential use of the proposed approach. Furthermore in the specific context it is shown how the proposed methodology can be used as a tool for assessing a questionnaire's reliability and construct validity.

Methods

Structures construction

In this section we briefly describe how existing FCA concept lattice construction algorithms have been applied for deriving the knowledge structure delineated by a given (conjunctive) *skill map* (Q, S, f). A knowledge state K is said to be delineated by a subset $X \subseteq S$ of skills via the *conjunctive model* if

$$K = \{q \in Q : f(q) \subseteq X\} \tag{6}$$

The basic idea behind equation (6) is that an item can be solved by a set X of skills if all skills $f(q)$ needed by that item are contained in X. Therefore, the collection of all items that are solvable by X is the knowledge state K. Then the resulting family of all such states is the knowledge structure delineated by (Q, S, f) via the conjunctive model (Doignon & Falmagne, 1999).

Two remarks are needed. The first one regards the fact that the "skills" used in the present article are not considered in the sense of "abilities needed to solve a particular sub-set of items", but in the one of "set of diagnostic criteria satisfied by a patient who answers "True" to a particular sub-set of items". Therefore the set S contains diagnostic criteria rather than skills. The second remark is referred to the use of the conjunctive model to depict the relations among sets of attributes and set of items answered. In the conjunctive model it is hypothesized that a subject who responds to an item would present all the attributes investigated by the item. Another possible approach to this issue is the *disjunctive model* (Doignon & Falmagne, 1999). Using this model the answer to an item implies that the subject displays at least one of the attributes investigated by that item. It is easy to understand how, using this model, a different perspective is taken in looking at the clinician's interpretation of the score obtained by a patient.

It is now important to highlight a fundamental difference between the two models. Using the conjunctive model, affirmative answers are more informative than negative ones. In fact, each affirmative answer indicates that the patient has all the attributes implied by the item. On the other hand, in the disjunctive model, negative answers are more informative than the affirmative ones because they indicate that the patient does not have any attribute implied by the item. This consideration has important consequences on the practical side: it seems reasonable to use the conjunctive model when testing a clinical sample and a disjunctive model with a normal population. Although not done in this article, it would be interesting to assess the performance of the disjunctive model with the present data-set.

The conjunctive model has been chosen because it represents the clinician's interpretation of the score obtained to a questionnaire in the following sense: usually a clinician looks at the score obtained by the patient and evaluates whether it is clinically significant or not. In other words, he assumes that the patient displays all the characteristics investigated by the specific scale. Using the conjunctive model it is possible to more deeply investigate the relations between the items on the basis of the attributes that each item satisfies, assuming that whenever a subject answers "True" to an item he displays (or at least he supposes to possess) all the attributes investigated by that item.

The structure's construction algorithm rests on the following principles and concepts.

A formal context corresponding to (Q, S, f) can be derived by interpreting Q as the collection of objects, S as the collection of attributes, and by defining a binary relation $R \subseteq Q \times S$ so that, for all pairs $(q, s) \in Q \times S$

$$qRs \iff s \notin f(q) \tag{7}$$

With these basic definitions at hand the triple (Q, S, R) can be regarded as a formal context. According to equation 7 the notation qRs should be read as "skill s is not required by item q". As an effect of this definition, the intent $q' := \{s \in S \mid qRs\}$

is just the complement of $f(q)$ in S (see in this respect Doignon & Falmagne, 1999, p. 96). The collection $\mathcal{I}$ of all the intents of the concept lattice corresponding to this context could then be obtained by closing under intersection the collection $\{q' : q \in Q\}$ of all object intents. There are a number of different algorithms doing this task in a quite efficient way (Ganter & Wille, 1999, p. 64) and many different programs have been developed to implement these algorithms (Guénoche, 1990; Valtchev, Missaoui, Godin, & Meridji, 2002; Vogt & Wille, 1994). For any subset $X \subseteq S$ the corresponding extent X' is obtained by (3). It is easily seen that by (7) this collection can also be rewritten as

$$X' = \{q \in Q : f(q) \subseteq S \setminus X\}, \tag{8}$$

which, by (6), happens to be a knowledge state delineated by (Q, S, f). Then, it is a well-known fact in FCA that X'' (i.e. the set of attributes obtained by the application of (3) to X') $\in \mathcal{I}$ for all $X \subseteq S$. From this fact it follows that, indeed, $\{Y' : Y \in \mathcal{I}\}$ is the collection of *all* knowledge states delineated by the skill map at issue. To summarize, the construction procedure that has been used in practice was the following one:

1. after defining an appropriate skill map (Q, S, f) for the items at hand, the relation R corresponding to it was obtained by an application of the simple rule (7);

2. then the whole concept lattice corresponding to the context (Q, S, R) was

produced by means of the software GaLícía (Valtchev et al., 2002);

3. at this point the knowledge structure delineated by f was simply the collection of all the extents of the generated concept lattice.

The knowledge structure composed by the extents of the concept lattice (i.e. sets of items) is closed under set intersection. It is known (Doignon & Falmagne, 1985, 1999) that in this case each single item has a unique set of prerequisites. In the present article, and more generally in the analysis of a questionnaire, it is reasonable to accept this assumption. In fact, each item is defined by a set of attributes and, via the conjunctive model, it is assumed that a person answering positively to an item has all the attributes of that item. Thus, the set of attributes is unique and the minimal response pattern for each item is unique too. This consideration changes when applied to the whole diagnostic process, as it is not realistic to assume that a specific diagnosis is associated to a unique minimal set of symptoms.

The Maudseley Obsessive Compulsive Questionnaire

The short form of the Maudsley Obsessive-Compulsive Questionnaire (Sanavio & Vidotto, 1985) included in the assessment battery Cognitive Behavioral Assessment 2.0 (CBA 2.0; Bertolotti, Zotti, Michielin, Vidotto, & Sanavio, 1990) is composed of 21 dichotomous items (True-False) investigating the main characteristics of OCD. The questionnaire is subdivided into three sub-scales investigating three of the main specifications of the disorder.

The first sub-scale is called "Checking", it is composed of 8 items investigating some habits of controlling and re-controlling many things; for instance item 4 "I must check many times some particular things (e.g. gas or water taps, doors, etc.)", or item 14 "I usually check things more than once". The score ranges from 0 to 8 and the clinical cut-off is set at the 95° percentile.

The second sub-scale is called "Cleaning", it is composed of 9 items investigating the habits of washing, cleaning and sense of contamination; for instance item 3 "When I touch an animal I feel contaminated", or item 17 "Every morning I spend a lot of time in washing myself completely". The score ranges from 0 to 9 and the clinical cut-off is set at the 95° percentile.

Finally the third sub-scale is called "Doubting-Ruminating", it is composed of 4 items investigating the presence of intrusive and disagreeable thoughts, for instance item is 2 "I frequently have disagreeable thoughts and I cannot get rid of them". The score ranges from 0 to 4 and the clinical cut-off is set at the 95° percentile.

The items of the MOCQ-R have been constructed mostly by referring to the diagnostic criteria for the OCD included in a previous version of the DSM. In order to analyze the items of the MOCQ-R we used the latest version of the manual, the DSM-IV-TR (APA, 1995). The OCD is included in the category of "Anxiety Disorders" and its diagnosis is based on five main criteria. The first criterion has been further sub-dived into two main parts: the first one deals with "Obsessions" while the second one refers to "Compulsions".

In the present paper the diagnostic criteria of DSM-IV-TR have been used to analyze each single item of the three scales of the MOCQ-R. The result of such analysis is represented by three formal contexts (consisting of three Boolean matrices) where the objects of the context were the items of the sub-scale, while the attributes were the diagnostic criteria of the DSM-IV-TR for the OCD. The underlying relation (gIm) indicates that the item g investigates the criterion m. In the next subsection the details of the procedure are described. In particular the method of construction of the structures is described and some descriptive indications about attributes implications revealed by the structures are provided.

The Structures of the MOCQ-R

In Figure 1 the concept lattice (i.e. the knowledge structure having sets of items as states) obtained for the sub-scale "Doubting-Ruminating" is presented.

[INSERT FIGURE 1 ABOUT HERE]

A short explanation of this kind of figure is needed. Figure 1 displays the complete lattice obtained for the "Doubting-Ruminating" scale. Each node of the lattice represents a formal concept. The number assigned to each node is not important (in this article). In the present paper we are interested in the collection of objects (items) and attributes (diagnostic criteria) listed in each single gray rectangle. The prerequisite relation among items has to be read bottom-up in the figure. These general rules have to be applied to all the figures included in this paper.

Going into the details of Figure 1, it is interesting to note that there are three different paths from the empty set to the total set of items. More specifically, from the structure it emerges how Item 5 is a prerequisite for Item 2, in fact there is no state including Item 2 and not including Item 5. From the attributes point of view it appears that Item 2 has all the attributes of Item 5 plus some other attributes. In Figure 1 it seems that the intent of the set of objects $\{5, 2\}$ (see node 2 of Figure 1) contains less elements (i.e. B, Cb, CA1a, Ca, CA2a, CA2b, CA1c, Cd) than the one of the intent of Item 5 (see node 4 of Figure 1; i.e. B, Cb, CA1a, Ca, CA2a, CA2b, CA1c, Cd, OA1b, OA3). In fact the structure of Figure 1 is derived by the dual of the formal context having the four items of the sub-scale Doubting-Ruminating as objects, and the diagnostic criteria of DSM-IV-TR as attributes, that is the attributes included in nodes 2 and 4 are not satisfied respectively by the sets of items $\{5, 2\}$ and $\{5\}$.

In Figure 2 the concept lattice obtained for the sub-scale "Checking" is presented.

[INSERT FIGURE 2 ABOUT HERE]

Given this structure it seems that Item 15 ("I follow a very precise order in everything I do") is a prerequisite for most items, and this indicates that the attributes present in it are replicated in many other items of the sub-scale.

The remarks on the "Cleaning" structure will be presented later in the "Results" section because some further elements have to be introduced to understand the steps followed to derive the final structure.

Testing the Structures

Knowledge structures like the ones presented above are by definition deterministic, they represent a model of possible response patterns of a sample of subjects, but they do not predict the probability of each pattern. As suggested by Doignon and Falmagne (1999) there are two main reasons to introduce probabilities in the model: the first one is that each state should be present with different frequencies in the population; the second one is that the observed response pattern of a subject could not represent his/her real knowledge. From the second reason the opportunity to consider two parameters related to each item follows: the "careless error" (also called "false negative"; α) and the "lucky guess" (also called "false positive"; β) represent respectively the probability that a subject does not solve an item that he is able to solve and the probability of solving an item that he is not able to solve. In other words it is reasonable to introduce conditional probabilities of responses given the knowledge states.

Doignon and Falmagne define a *probabilistic knowledge structure* as a triple $(Q, \mathcal{K}, p)$

where: i)$(Q,\mathcal{K})$ is a knowledge structure; ii) p is a probability distribution on $\mathcal{K}$. In the model at issue, given a state, the responses to the items are locally independent. Thus, starting from the probabilistic knowledge structure $(Q,\mathcal{K},p)$, given a specific response pattern $R \subseteq Q$ we will define a function $s : (R,K) \mapsto s(R,K)$ assigning to each response pattern its conditional probability given that a subject is in state K (for all states $K \in \mathcal{K}$), the *response function* for the probabilistic knowledge structure. Thus, we obtain for each response pattern a probability distribution:

$$p(R) = \sum_{K \in \mathcal{K}} s(R,K)p(K) \tag{9}$$

Since the response function s satisfies local independence for each item $q \in Q$, the conditional probability $s(R,K)$ is determined given the two probabilities α and β respectively the careless error and lucky guess related to each item q. Formally:

$$s(R,Q) = (\prod_{q \in K \setminus R} \alpha_q)(\prod_{q \in K \cap R} (1-\alpha_q))(\prod_{q \in R \setminus K} \beta_q)(\prod_{q \in \overline{R \cup K}} (1-\beta_q)) \tag{10}$$

Equation 10 represents the so called *basic local independence model* (BLIM), which is used in the present article.

The parameters of the model have been estimated by the Expectation-Maximization Algorithm (Dempster, Laird, & Rubin, 1977).

Results

In order to validate the obtained structures, a data-set provided by a sample of patients from the north eastern part of Italy (n = 33; age ranging from 19 to 43 years; 20 males, 13 females) with a diagnosis of OCD has been used. The parameters of the BLIM have been estimated for each of the three structures. The fit of each of the three models has been tested by Pearson's chi-square. It is well known that for large data matrices (as those used in the present study) the asymptotic distribution of χ^2 is not reliable. Therefore a p-value for χ^2 has been obtained by parametric bootstrap(n. of replications$= 5,000$).

In the first part of the analysis we tested, for the sub-scale "Cleaning" (9 items), a knowledge structure composed by 80 knowledge states derived by the closure under intersection of the intents of the formal context. The single items' α and β parameters seem quite small for almost all items (Table 1).

[INSERT TABLE 1 ABOUT HERE]

As previously discussed, in KST α and β represent the probability of "careless errors" and "lucky guesses" respectively. It seems more appropriate, in the present paper, to refer to them as "false negative" and "false positive" in their clinical acceptation.

The results of the bootstrap performed on this model do not support the goodness of fit of the structure ($\chi^2 = 179.98; p = 0.0487$). By the analysis of the content of each single item the deletion of Item 1 appeared to be the best solution to the poor fit. In fact it says "I do not use the public phone because I am afraid of possible contaminations", which seems rather obsolete. The deletion of this item reduces the number of possible states to 40.

[INSERT FIGURE 3 ABOUT HERE]

The new structure (i.e. without item 1) has been tested together with the ones obtained for the other two sub-scales. Results show good fit indexes for all three models. Table 2 displays the global fit indexes obtained for the three models along with corresponding p-values obtained by parametric bootstrap.

[INSERT TABLE 2 ABOUT HERE]

The p-value of the bootstrap performed on the sub-scale “Cleaning” needs some further explanation. The small value observed could be explained by the fact that the number of states derived by closure under intersection of the formal context is about 40. This is due to the fact that in this sub-scale the items are rather heterogeneous. They investigate a number of attributes higher than the one investigated by the items of the other sub-scales. The number of states found for the scale “Doubting-Ruminating” is 6 and the number of states found for the sub-scale “Checking” is 19. A larger sample could improve the level of the p-value also because the levels of α and β are good especially for the “Cleaning” sub-scale.
Table 3 shows the α and β parameters obtained for each item in the test.

[INSERT TABLE 3 ABOUT HERE]

The overall model fit along with the results displayed in Table 3 indicate that the three models quite accurately depict the relations between different items of the MOCQ-R. These relations are well represented by the formal context built using the items of MOCQ-R as objects and the diagnostic criteria of DSM-IV-TR for the OCD as attributes. The two critical parameters, i.e. the high values of "false positive" estimates for items 2 and 12 can be explained in two different but not necessarily exclusive ways: the first one refers to the small number of subjects composing the sample. The second one relates to the fact that the items are clinical, thus a sort of misinterpretation of the meaning of the items is possible. This fact can be better understood by looking at the text of the items. Item 2 says "I frequently have disagreeable thoughts and I cannot get rid of them". This item is composed by two separate sentences: the first one is "I frequently have disagreeable thoughts", the second one is "I cannot get rid of them". In the conjunctive model a subject is supposed to provide a positive answer when he has all the elements required. In this case the subject may have answered "True" even if he believed that only one of the two sentences was true. Item 12 says "One of the greatest problems of mine is the repeated check of things". In this case some problems may arise from the interpretation of either "greatest" or "repeated". Anyway, since the analyzed questionnaire is clinical, it seems reasonable to expect higher α and β values than in the classical field of application of KST. In fact a subject could intentionally fake the answer. Furthermore the subject's answer could be affected by his poor introspection capabilities.

Discussion

The present article provides some interesting results both from the clinical and the methodological point of view.

From the methodological point of view, the proposed analysis can be used as a tool to validate the construct and content validity of a given questionnaire. In the presented example the validity of MOCQ-R has been assessed on the basis of the presence-absence of the criteria of the DSM-IV-TR which gives a theoretical-clinical interpretation of the questionnaire. The joint application of FCA and KST can evaluate whether a questionnaire actually measures the underlying construct (in this case the Obsessive-Compulsive Disorder). In the example, the underlying construct was the set of diagnostic criteria for the OCD. In this perspective the validity analysis rests on verifying the relation between the items and the criteria (i.e. the attributes of the formal context).

In typical applications a clinician uses the questionnaire by considering the score obtained by the patient. Since the underlying construct is multidimensional, the mere score of a patient is a dramatic reduction of the potential information provided by the test. By the proposed approach the information collected by the questionnaire can be used to point out differences between patients that otherwise would be hidden by the simple score. Indeed, from the clinical perspective, the proposed methodology could be regarded as an in depth evaluation of the construct investigated by the questionnaire. The response patterns corresponding to clinically significant scores (i.e. $> 95°$ percentile) could point to different collections of diagnostic symptoms, and these differences are not captured by the simple score. For instance it is possible to note that the two collections of items $\{3, 8, 10, 13, 16, 18, 20\}$ and $\{3, 8, 13, 16, 17, 18, 20\}$ (corresponding to nodes 2 and 4 of Figure 3 respectively) are equally scored 7, but they correspond to different collections of attributes. This means that patients obtaining the same score might have different sets of symptoms.

In the presented case, the relation between the items of the MOCQ-R and the criteria of the DSM-IV-TR has been constructed assuming a conjunctive model. As discussed in the "Structure Construction" section this assumption seems reasonable when a sample of clinical patients is assessed. There are other situations in which the disjunctive model could be more appropriate, for instance when reasoning in terms of a whole diagnostic process in a wider spectrum situation (i.e. a complete battery for clinical assessment).

Considering the practical application of the approach proposed in this article, in a clinical setting the relations found between items could be used to calibrate an algorithm for the adaptive and efficient evaluation of patients. KST has been developed with the aim to construct an efficient tool for assessing knowledge. Some computerized algorithms have been developed with this aim (Falmagne & Doignon, 1988a, 1988b; Dowling et al., 1996; Hockemeyer et al., 1998). These algorithms can be easily adapted to the clinical context for which the structures of figures 1, 2 and 3 have been developed.

A final remark concerns the α and β probabilities of the items. Since they are interpreted as false positives and false negatives, these probabilities are clearly expected to be small. Large values of these parameters could point to bad specification of the model or to bad wording of the items. Therefore these parameters can be used as a diagnostic tool for improving the model or the items. For instance, when a large value of the parameter α is observed for a given item, there might be one or more attributes both necessary to positively answer to a specific item and not included in the model for that item. In other words even if in a given model the attributes referred to a specific item i are for instance $\{a, b, c\}$ a large value of α indicates that one or more further attributes have to be displayed by a subject to answer i. There are two main ways to cope with this problem: the first one is to reformulate the model with respect to i, including the necessary attributes; the second one is to reword the specific item so that it is in accord with the model. Working with standardized questionnaires the first solution might be preferred, but when a questionnaire is under construction the second approach could be the most interesting one.

Some different issues have to be considered when coping with a large value of β. There are two main explanations of this. The first one is that in the model one or more attributes have been wrongly considered necessary to answer "True" to a specific item. In this case it is possible to eliminate from the set of the attributes of that item all those attributes that are not necessary for it; otherwise an alternative could be the rewording of the item. The second explanation is that the conjunctive model is not a good representation of the relation between items and attributes. In this case the possibility to apply a disjunctive model can be considered.

In conclusion, the present paper highlights how the proposed application to a clinical context of KST and FCA seems to be a useful and fruitful research and clinical perspective. It has to be further investigated (using larger samples and different clinical disorders) whether the proposed methodology would be able to provide the clinician with the opportunity to perform a more in depth analysis of patients' responses. This opportunity will allow the clinician to do personalized diagnosis able to pinpoint subject specific characteristics. In the long term this methodology will allow the construction of an adaptive, efficient and effective tool for psychological diagnosis. This instrument will be very similar to the instruments already built for knowledge assessment.

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References

Albert, D., Schrepp, M., & Held, T. (1992). Construction of knowledge spaces for problem solving in chess. In G. H. Fisher & D. Laming (Eds.), *Contributions to Mathematical Psychology, Psychometrics, and Methodology* (pp. 123–135). New York: Springer-Verlag.

American Psychiatric Association [APA]. (1995). *DSM-IV-TR. Diagnostic and Statistical Manual of Mental Disorders* (Fourth Edition, Text Revision ed.). Washington D. C.: American Psychiatric Association.

Bertolotti, G., Zotti, A. M., Michielin, P., Vidotto, G., & Sanavio, E. (1990). A computerized approach to cognitive behavioural assessment: An introduction to CBA-2.0 primary scales. *Journal of Behavior Therapy and Experimental Psychiatry*, *21 (1)*, 21–27.

Birkhoff, G. (1937). Rings of sets. *Duke Mathematical Journal*, *3*, 443–454.

Birkhoff, G. (1967). *Lattice Theory.* Providence, R.I.: American Mathematical Society Colloquium Pubblication. (No. XXV)

Dempster, A. P., Laird, N. M., & Rubin, D. B. (1977). Maximum likelihood from incomplete data via the EM algorithm. *Journal of the Royal Statistical Society. Series B (Methodological)*, *39 (1)*, 1–38.

Doignon, J. P., & Falmagne, J. C. (1985). Spaces for the assessment of knowledge. *International Journal of Man-Machine Studies*, *23*, 175–196.

Doignon, J. P., & Falmagne, J. C. (1999). *Knowledge Spaces.* Berlin Heidelberg: Springer-Verlag.

Dowling, C. E., Hockemeyer, C., & Ludwig, A. H. (1996). Adaptive assessment and training using the neighbourhood of knowledge spaces. In C. Frasson, G. Gauthier, & A. Lesgold (Eds.), *Intelligent Tutoring Systems* (pp. 578–586). Berlin: Springer-Verlag.

Falmagne, J. C., & Doignon, J. P. (1988a). A class of stochastic procedures for the assessment of knowledge. *British Journal of Mathematical and Statistical Psychology*, *41*, 1–23.

Falmagne, J. C., & Doignon, J. P. (1988b). A markovian procedure for assessing the state of a system. *Journal of Mathematical Psychology*, *32*, 232–258.

Falmagne, J. C., Koppen, M., Villano, M., & Doignon, J. P. (1990). Introduction to knowledge spaces: how to build, test and search them. *Psychological Review*, *97*, 201–224.

Ganter, B., & Wille, R. (1999). *Formal Concept Analysis: Mathematical foundations*. Berlin: Springer-Verlag.

Guénoche, A. (1990). Construction du treillis de Galois d'une relation binaire. *Mathmatiques et Sciences Humaines*, *109*, 41–53.

Hockemeyer, C., Conlan, O., Wade, V., & Albert, D. (2003). Applying competence prerequisite structures for e-Learning and skill management. *Journal of Universal Computer Science*, *9*, 1428–1436.

Hockemeyer, C., Held, T., & Albert, D. (1998). Rath- a relational adaptive tutoring hypertext www-environment based on knowledge space theory. In *Procedings of the CALISCE '98 meeting (Computer Aided Learning in Science and Engineering).* Göteborg, Sweden, June 15-17, 1998.

Hodgson, R. J., & Rachman, S. J. (1977). Obsessional-compulsive complaints. *Behaviour Research and Therapy*, *15*, 389–395.

Lukas, J., & Albert, D. (1993). Knowledge assessment based on skill assignement and psychological task analysis. In G. Strube & K. F. Wender (Eds.), *The Cognitive Psychology of Knowledge* (pp. 139–159). Amsterdam: Elsevier.

Rusch, A., & Wille, R. (1996). Knowledge Spaces and Formal Concept Analysis. In H. H. Bock & W. Polasek (Eds.), *Data analysis and information systems. Statistical and conceptual approaches* (pp. 427–436). Berlin: Springer-Verlag.

Sanavio, E., & Vidotto, G. (1985). The components of the Maudsley Obsessional-Compulsive Questionnaire. *Behaviour Research and Therapy*, *23*, 659–662.

Spoto, A., Vidotto, G., Postal, D., & Pendoni, E. (2008). Rappresentazione formale dell'assessment cognitvo-comportamentale per il disturbo ossessivo-compulsivo. *Giornale Italiano di Medicina del Lavoro ed Ergonomia - Supllemento B, Psicologia*, *30*(3), B91–B100.

Valtchev, P., Missaoui, R., Godin, R., & Meridji, M. (2002). Generating frequent itemsets incrementally: two novel approaches based on Galois lattice theory. *Journal of Experimental and Theoretical Artificial Intelligence*, *14*(2-3), 115–142.

Vogt, F., & Wille, R. (1994). Toscana - a graphical tool for analyzing and exploring data. In R. Tamassia & I. G. Tollis (Eds.), *Graph drawing, volume 894 of lecture notes in computer sciences* (pp. 226–233). New York: Springer-Verlag.

Wille, R. (1982). Restructuring lattice theory: An approach based on hierarchies of concepts. In I. Rival (Ed.), *Ordered Sets* (pp. 445–470). Dordrecht: Reidel.

Table 1:

Estimated parameters α and β for each item of the "Cleaning" scale

Item	1	3	8	10	13	16	17	18	20
α	0.00	0.26	0.11	0.00	0.25	0.05	0.00	0.16	0.00
β	0.00	0.00	0.00	0.31	0.35	0.00	0.00	0.00	0.00

Table 2:

The global fit indexes of the three models

Model	n. of states	*df*	χ^2	bootstrap p
Checking	18	222	127.39	0.2186
Cleaning	42	198	141.65	0.1003
Doubting-Ruminating	6	2	2.30	0.8056

Table 3:

Estimated parameters α and β for each item of the three sub-scales

Checking			Cleaning			Doubting-Ruminating		
Item	α	β	Item	α	β	Item	α	β
4	0.00	0.00	3	0.27	0.07	2	0.00	0.40
7	0.24	0.23	8	0.21	0.00	5	0.06	0.00
9	0.06	0.13	10	0.00	0.21	6	0.22	0.00
11	0.00	0.26	13	0.23	0.27	21	0.10	0.09
12	0.00	0.40	16	0.05	0.00			
14	0.00	0.00	17	0.00	0.00			
15	0.16	0.00	18	0.24	0.00			
19	0.00	0.20	20	0.16	0.00			

Figure 1: The Concept Lattice obtained for the sub-scale "Doubting-Ruminating"

Figure 2: The Concept Lattice obtained for the sub-scale "Checking"

Figure 3: The Concept Lattice obtained for the sub-scale "Cleaning" excluding Item 1

Figure 1:

The Concept Lattice obtained for the sub-scale “Doubting-Ruminating”

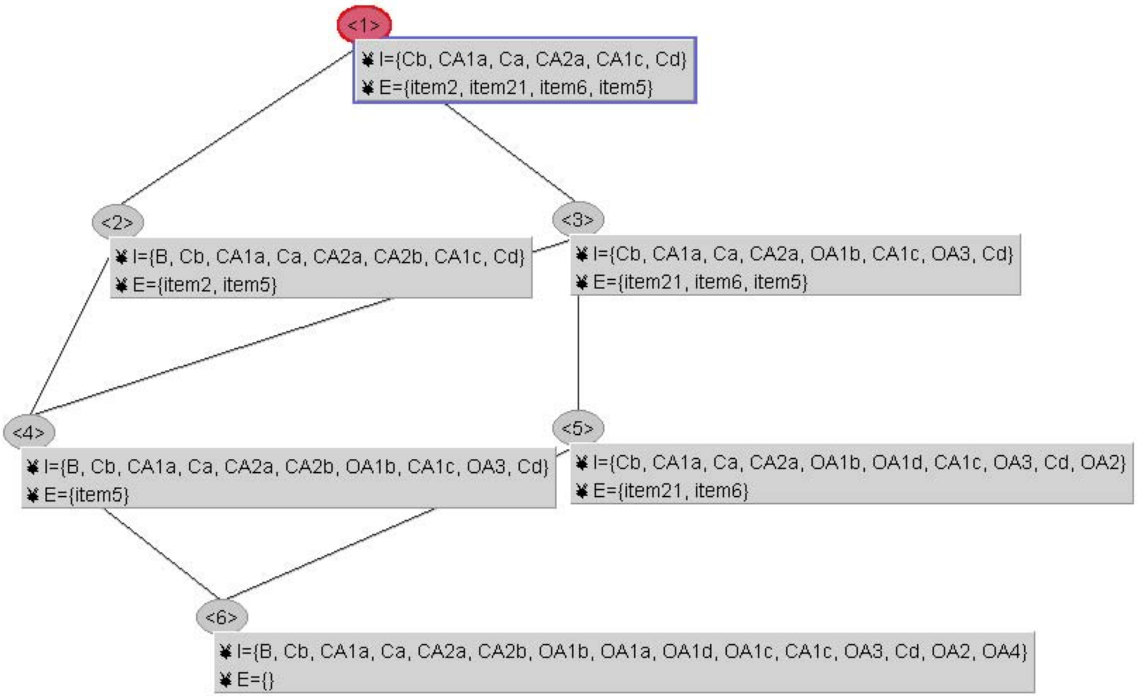

Figure 2:

The Concept Lattice obtained for the sub-scale “Checking”

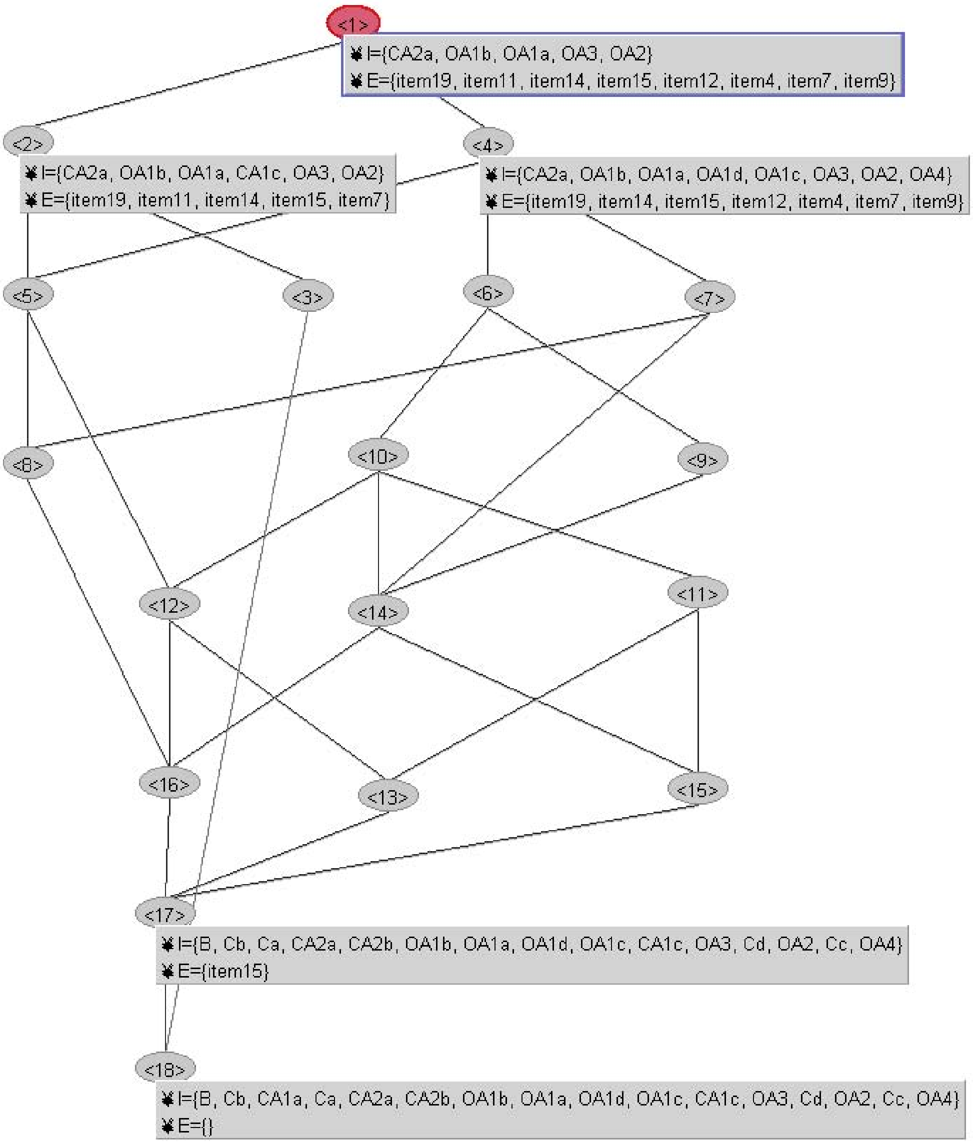

Figure 3:

The Concept Lattice obtained for the sub-scale “Cleaning” excluding Item 1